Solutions to Thompson's Lamp Paradox by Physical Planck Limitations

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***Abstract*—In order to solve Thompson's lamp paradox, I impose physical Planck-scale limitations on the experiment.**

I. Introduction

IN 1954, British philosopher James F. Thompson created a variation of one classic Zeno paradox. His goal? Understanding the supertask, i.e. completing an infinite number of tasks in a finite amount of time

The paradox can be stated as follows: consider a lamp with an on/off switch. First, turn the lamp on. After one minute, turn it off. After 1/2 min, turn it on, and after 1/4 min, turn it off again. Continue toggling the lamp on and off, each time waiting half the time you had previously. At the end of 2 minutes, is the lamp on or off?

To answer this question, we go about seeking a solution by imposing two physical limitations on the system:

1) Nothing can travel faster than the speed of light
2) Space ceases to have meaning below the Planck length.

II. Planck limitations

By combining the fundamental constants $\hbar, G,$ and c in a particular way, Max Planck was able to calculate both a length and time involving only fundamental constants —things we measure, not create like "meters." These two values are interpreted as such: physical space ceases to have meaning at scales below the Planck length. In this regime, quantum fluctuations dominate. The Planck time, is the time it takes a photon (traveling at velocity c) to traverse the Planck length.

III. How to Solve

To solve, I pose the question: *After how many toggles would the time waited to the next toggle be less than the Planck time?* This led me to the inequality:

$$\frac{t}{2^n} \leq t_p \tag{1}$$

where t is time, t_p is the Planck time, and n is the number of toggles. This can be solved for n and rewritten as:

$$n = \log_2\left(\frac{t}{t_p}\right) \tag{2}$$

I decided to solve this by writing by writing a function in MATLAB.

Advisor: Keith McPartland, "Paradoxes." Williams College 2015

IV. Matlab Script

```
function [lampState] = lampState (time)
planckTime = 5.3911613 × 10^-44;
n=log2(time/planckTime);
nRounded = ceil(n); % rounds n up to
nearest integer;

% 1=true="on";
% 0=false="off";
if mod(nRounded, 2)==0 % if even # then lamp is on
  lampState="on";
else
  lampState="off";
end
```

V. Data

TABLE I
FINAL LAMP STATES

Time (sec)	Final State
120	off
100	off
80	off
60	on
40	on
20	off

VI. Results

By imposing Planck scale limitations to the Thompson lamp paradox, we can see that after two minutes the lamp will on. Remember that this is in the condition where at $t = 0\ sec$ the lamp is on, and at $t = 60\ sec$ the lamp is off. We can also see from the data that the final state of the lamp depends on your time constraint (obviously). It would be interesting to determine that lamp state as a function of starting time and plot this.

VII. Furthermore

Using the Planck scale restrictions, we find the lamp is on at the end of 2 minutes. This restriction if of course highly impractical. It would be modeled by two perfectly-reflective mirrors bouncing a single photon back and forth, where the starting distance of the mirrors $1\ light - minute$ and halved every bounce. An interesting follow up would be to impose a practical limitation on the speed of communication between the lamp and the switch.

References

[1] I first learned about this paradox while taking Philosophy of Paradoxes with Professor Keith McPartland